

RECENT ADVANCES IN CONTACT MECHANICS

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Description

Phenomena of contact between deformable bodies or between deformable and rigid bodies abound in industry and everyday life. Contact of braking pads with wheels, tires with roads, pistons with skirts are just a few simple examples. Common industrial processes such as metal forming, metal extrusion, involve contact evolutions. Owing to their inherent complexity, contact phenomena are modelled by nonlinear evolutionary problems that are difficult to analyze.

Considerable progress has been achieved recently in modelling, analysis and numerical simulations of various contact processes. Indeed, a general Mathematical Theory of Contact Mechanics is currently emerging. It is concerned with the mathematical structures which underlie general contact problems with different constitutive laws, i.e., materials, varied geometries, and different contact conditions. The aim is to provide a sound, clear and rigorous background to the following:

- construction of models based on thermodynamic principles which are motivated by applications;
- establishing the existence of solutions;
- proving the uniqueness of the solutions, or establishing their nonuniqueness and finding criteria for choosing the appropriate solution;
- investigating the stability of solutions and their asymptotic behavior;
- analysis of approximation schemes including convergence results of the discrete solutions and optimal error estimates;
- construction of reliable and efficient algorithms for the numerical approximations of the models with guaranteed convergence.

The Theory also provides an environment or a structure where a number of engineering problems can be investigated and reliable results found.

The aim of this course is to provide the state of the art of the Mathematical Theory of Contact Mechanics and to present some of its applications in engineering. The lectures will provide treatment of contact problems by presenting arguments and results in modeling, analysis and numerical simulations.

The presentation is divided into three lectures, as follows :

- *Part I : Modelling of Contact Processes.*
- *Part II : Variational Formulations and Numerical Simulations.*
- *Part III : References, Comments and Conclusions.*

Part I concerns mainly the mechanical modelling of the contact problems including the basic equations, the constitutive relations and the frictional conditions. It also contains a description of the additional effects and phenomena that arise during the process of contact (thermal effect, piezoelectric effect, wear, adhesion, damage).

Part II starts with the variational formulations of various type of contact problems. The importance of this step in arise from the fact that in general, the classical formulations of contact problems do not have any solution. Therefore, in order to provide a result concerning the well-posedness of the models, there is a need to reformulate the contact problems in a weaker sense, i.e. to derive their variational formulation. We continue this part by presenting numerical simulations and applications to various engineering problems involving contact processes.

Part III contains a short summary of the results, bibliographic comments, and a discussion of additional problems and future directions. We provide references to where the principal results presented can be found, as well as comments on important topics related to, but not included, in *Parts I and II*. Finally, we discuss related issues, describe problems which have to be resolved to facilitate further growth of the theory, and present our conclusions.

The lectures are accessible for engineers and scientists, as well as advanced graduate students.

References

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- [2] Shillor M., Sofonea M. and Telega J.J., *Models and Analysis of Quasistatic Contact*, Lecture Notes in Physics **655**, Springer, Berlin Heidelberg 2004.
- [3] M. Sofonea, W. Han and M. Shillor, *Analysis and Approximation of Contact Problems with Adhesion or Damage*, Pure and Applied Mathematics **276**, Chapman-Hall/CRC Press, New York, 2006.
- [4] M. Sofonea and A. Matei, Variational Inequalities with Applications. A Study of Antiplane Frictional Contact Problems, *Advances in Mechanics and Mathematics, Vol. 18*, Springer, New York, 2009.